Supplementary Notes

The Golden Section

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Collage and Montage in Golden Ratios
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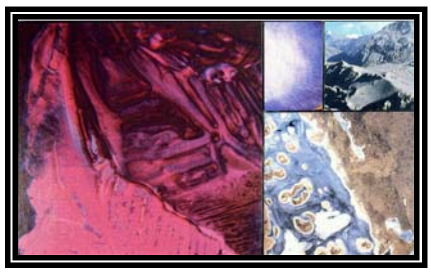


CORE New Art Space *2045 Larimer Street, Denver CO 80205*

The Golden Ratio

Knowledge of the Golden Section, ratio, or proportion has been known for a very long time. The Egyptians knew about it and the Greeks learned about it from them. It is called *phi*, ***Φ***, in honor of Phideas, the architect of the Parthenon and is approximated by the irrational fraction 0.618034... Shamans, priests, and artists throughout the world and across history have understood and applied ***Φ*** to ritual, architecture, art, and the crafting of musical instruments and everyday objects.

Φ shows up throughout nature. Recall the famous drawing by Da Vinci showing man within the circle and the Golden Ratios in the human body, and more recently, Le Corbusier's *The Modular*. For example, the finger bones are in ***Φ*** ratio to each other, and the position of features on the human face follow ***Φ***. The major 6th harmony interval in music is in ***Φ*** ratio to the octave.

The Golden Rectangle (GR), the organizing form in my current work, is a rectangle with a short to long side ratio of 1: 1.618, or 1: *(1 +* ***Φ****)*. An interesting property of GR's is that if you cut out a square starting from one of the short sides of the GR, you will be left with another GR. You can continue to cut out short side squares for each successively smaller GR and another smaller GR will remain. And the dimensions of each successively smaller rectangle will be in ***Φ*** ratio to the previous larger size. A series of my collage explore this ***Φ***-ratio "coiling" property of GR's, seen here in Figure 1:

Figure 1

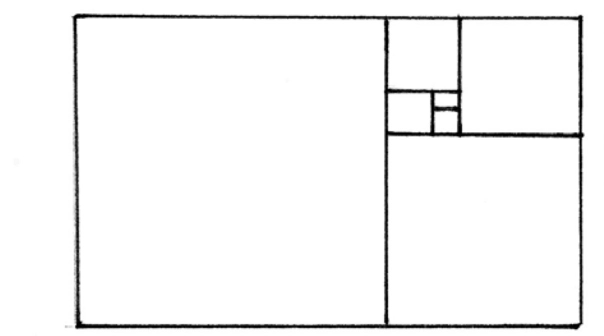
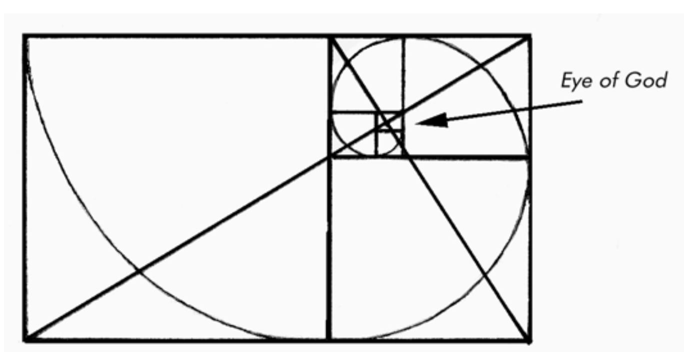


Figure 2 shows a logarithmic spiral superimposed on a coiled GR. This study shows the ***Φ***-ratio sectioning of the GR with short side squares and the diagonals of the original seed GR (GR_0) - outside boundary lines) and the diagonal of the first ***Φ***-sectioned GR (GR_1). Note that the two diagonals intersect at a point called the "Eye of God," the origin of the logarithmic spiral.

Figure 2



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Logarithmic spirals are natural forms (remember the chambered nautilus?) and many natural forms will fit neatly with a GR. Examples include bird eggs, human heads, and spruce trees. So, it would seem that the distinction between a simple "geometric" form like a GR and an organic form like a logarithmic spiral is superficial. Both forms imply and can be derived from the other!

The symmetry of GR's means that the coiling can proceed in 4 different ways, explored in Figure 3. Here we can see that there are a lot of implied lines and shapes within a GR - and that there are four Eyes of God that are connected by another GR (shaded). The sides of the inner Eyes of God GR have a special ratio to the sides of GR_0 - it has length and width that are in square-root of 5 (1: 2.236...) ratio to the original seed GR_0 sides. The also irrational square-root of 5 appears often, both in the calculation of Φ (see below) and when GR forms are combined.

Figure 3

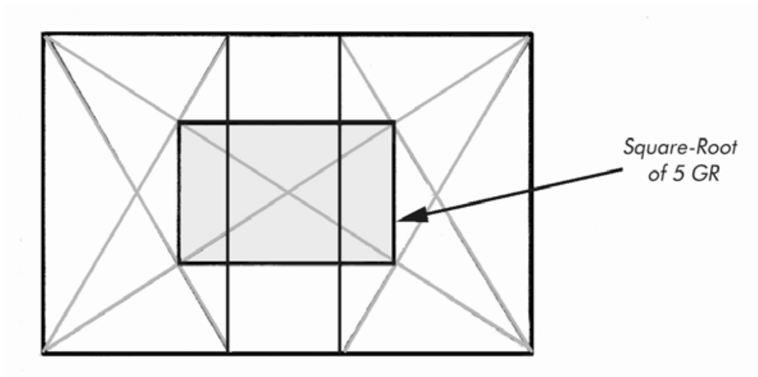
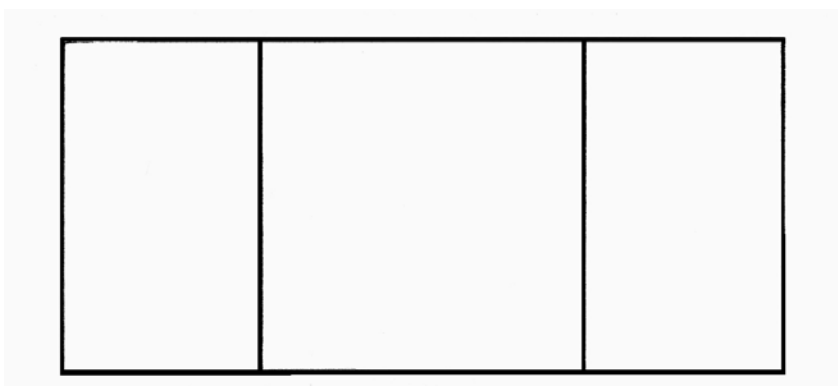


Figure 4 shows overlapping GR's which I use in a series of collage and mandala square collages in my recent work. Overlapping GR's that share a middle square are in the aspect ratio of 1: (square root of 5).

Figure 4



A final note about the GR involves *fractals*. Fractals are geometric forms that look the same no matter what the size scale. They are composed of repeating units that combine to make larger and similar units at larger scales. This property is called *self-similarity*, and it is also a huge property of nature and natural forms. Fractals also have a property called *fractional symmetry,* which means that the self-similar units are in non-integer fractional proportion to each other. The square is the self-similar shape that is repeated in the GR, and Φ is a non-integer proportionality ratio, so GR's qualify as basic fractals. All of the abstract images in my collages also contain fractal form elements, for example, coastlines and rock fractures have a fractal structure. I guess I am eat up with it!

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*Deriving and Calculating **Φ***

There are 2 general ways to derive ***Φ***. One approach uses the Fibonacci numbers, and the other is geometrical and algebraic. Fibonacci numbers are easier to comprehend, so let's start there.

Fibonacci Numbers: Fibonacci was the pen name of Leonardo of Pisa, a 13th century mathematician whose book, *Liber Abaci*, introduced western civilization to Arabic numerals (replacing Roman numerals), and a special sequence of numbers named after him. Fibonacci raised rabbits and observed their population numbers over successive generations. They increased in a peculiar "additive" way, and from this he surmised the more abstract number sequence. Starting with 0 and 1 as the first two numbers in the sequence, each successive number is determined by adding the previous two numbers. Starting with 0 and 1, the series goes like this:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

It turns out that this sequence of integers is much more than an arithmetic game. These numbers turn up *all the time* in nature, and are observed in the dimensions and branching of all plants, animals, as well as crystals. This happens because when things grow, they often grow on top of a previous structure, so that the new growth is "added to" the existing structure (like new offspring are added to the existing population). Many plants exhibit Fibonacci numbers in branching, and in spiral structures like the arrangement of rows of bracts on pinecones, petals on an artichoke, and scales on a pineapple.

This would just be a logical curiosity about growing things until we start calculating the *ratio* between adjacent Fibonacci numbers. Table 1 lists the Fibonacci numbers in the left column, the ratio calculations in the middle column, and the results in the right column. It turns out that after the fifth Fibonacci number, the ratio begins to get very close to the algebraic solution for ***Φ*** = 0.6180339. If you figure that two decimal places are as much as even the most careful craftsman can measure and cut, then the 6th Fibonacci ratio, 8 ÷ 13, will do as an approximation of 0.62 for ***Φ***. After the 16th Fibonacci number, the ratio approximates ***Φ*** to 6 decimal places. Higher Fibonacci number ratios yield changes in only in the 7th decimal place and beyond.

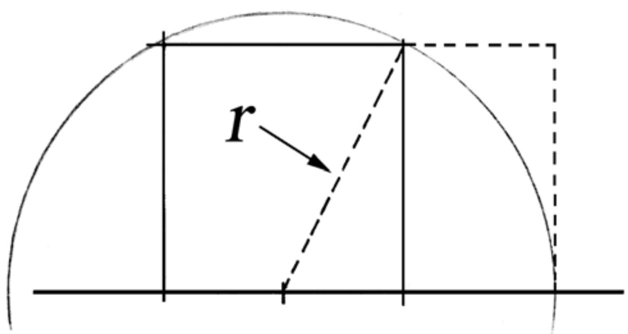
Table 1 Ratios between adjacent numbers in the Fibonacci series start to approximate the Golden Ratio, ***Φ***.

Fibonacci number	***Ratio Calculation***	***Result***		***Fibonacci number***	***Ratio Calculation***	***Result***
1				55	34 ÷ 55	0.618181
1	1 ÷ 1	1.000000		89	55 ÷ 89	0.617977
2	1 ÷ 2	0.500000		144	89 ÷ 144	0.618055
3	2 ÷ 3	0.666666		233	144 ÷ 233	0.618025
5	3 ÷ 5	0.600000		377	233 ÷ 377	0.618037
8	5 ÷ 8	0.625000		610	377 ÷ 610	0.618032
13	8 ÷ 13	0.615384		987	610 ÷ 987	0.618034
21	13 ÷ 21	0.619047		1,597	987 ÷ 1,597	0.618033
34	21 ÷ 34	0.617647		2,584	1,597 ÷ 2,584	0.618034

So, Fibonacci numbers are related to ***Φ*** and can be used to derive the Golden Ratio. I use frame dimensions of 21" x 13" for coiled GR collages (Figure 1) which are adjacent Fibonacci numbers. Saves a lot of trouble trying to get precise odd fractions of an inch cut by the frame cutter!

Geometric Method: The geometric approach to drawing a GR is fairly simple using a straightedge and compass. Refer to Figure 5. Basically, draw a straight base line across the bottom of a sheet. Draw a fairly large square near the middle of the line segment. Make sure the sides are perpendicular and equal. Find the midpoint of the bottom side of the square and mark it. Take the compass and stick the sharp point at the midpoint of the bottom of the square. With the compass point at the marked midpoint, open the compass until the pencil point touches the *upper right hand corner* of the square, radius ***r*** in figure 5. Now draw a big half-circle that intersects the baseline. Draw a line extending the square and a perpendicular line that meets the point where the big circle intersected the baseline.

Figure 5



Algebraic Method: For the algebraic method (which can be skipped if you are not mathematically inclined), consider the geometric definition of ***Φ*** on a line segment, *AB*, seen in Figure 6. We want to find the Golden Section, the point on *AB*, call it *C*, such that the ratio of the big chunk *AC* to the whole segment *AB* is the same as the ratio of the small chunk, *CB*, to the big chunk, *AC*. For *C* to be the Golden Section point, (*AC* divided by *AB*) must equal (*CB* divided by *AC*).

Figure 6

```
|<----------------AB------------------>|
A                       C              B
.______________________.______________.
|<----------AC--------->|<-----CB----->|
```

To simplify matters (or to confuse the less mathematically gifted), we will do some substitutions and let the whole segment *AB = 1*, and we will let *AC* be *phi,* ***Φ***. Now, it should be clear that the whole segment *AB* is the sum of the 2 parts, *AC* and *CB*, right? So we can set up an equation that says the same thing:

$$AB = AC + CB$$

and subtracting *AC* from both sides gives us

$$AB - AC = CB$$

and substituting *1* for *AB* and ***Φ*** for *AC*, we get

$$1 - \boldsymbol{\Phi} = CB$$

Figure 7 shows how the substitutions have changed the problem. Here we show the line segments with the new substituted terms.

Figure 7

```
|<--------------- 1 ---------------->|
A                      C             B
.______________________.______________.
|<---------- Φ -------->|<---(1 - Φ)-->|
```

With me so far? OK...now we are going to set up an algebraic equation to solve for ***Φ***. Remember our definition: for *C* to be the Golden Section of segment *AB*, the ratio of the big chunk *AC* to the whole segment *AB*, has got to be the same as the ratio of the little chunk *CB* to the big chunk *AC*. So now we can set this up as an equation:

$$\frac{AC}{AB} = \frac{CB}{AC}$$

OK... So now let's substitute 1 for *AB, **Φ*** for *AC*, and *(1 - **Φ**)* for *CB,* and we get:

$$\frac{\phi}{1} = \frac{1-\phi}{\phi}$$

We are almost there! Now, we need to "multiply the extremes and means" to simplify this equation and solve for ***Φ***. Looking at the equation like a box, the extremes are the upper left and lower right terms, the means are the lower left and upper right terms. Multiplying these gives you:

$$\phi^2 = 1 - \phi$$

Subtracting *(1 - **Φ**)* from both sides of the equation gives us the *characteristic equation* for solving for ***Φ***:

$$\phi^2 + \phi - 1 = 0$$

The following two values will solve this equation:

$$\frac{-1-\sqrt{5}}{2} \quad and \quad \frac{-1+\sqrt{5}}{2}$$

We ignore the first solution because it is negative, and the length of a line segment can't be a negative number. So the value of ***Φ*** calculates to ***0.6180339...***, an irrational non-repeating digit ratio.

BIBLIOGRAPHY

These books have been helpful in my study of the Golden Proportion and form in nature and art. I recommend Garland's *Fascinating Fibonaccis* to anyone who would like a general overview. It's easy to understand and gives a lot of good visual examples. Teachers may want to consider Garland as an introduction and Runion's *The Golden Section* for a more math-oriented approach with problems in each chapter. Garland is suitable for middle school kids while Runion is high school algebra level. Let me know if you have recommendations for other books on the subject of sacred geometry, the Golden Proportion, or form-based aesthetics.

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Philosophy: These books are not specifically about the Golden Ratio, but provide integrative links to the ideas of harmony, physical law, and self-similarity. They have been a huge inspiration and source of insight to me.

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